A Multi-Model Fast Denoising Method Based On The Wavelet Transform Threshold Denoising

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***Abstract*—The biomedical signals are often corrupted by noise in their acquisition or transmission resulting in lower Signal to Noise Ratio (SNR), which brings problematic obstacles to successive biomedical signal processing. So suppressing noise and improving SNR effectively is an essential procedure and key issue in the research on biomedical signal processing. In this paper, we propose a novel multi-model fast denoising method based on the Wavelet transform threshold denoising. The proposed denoising scheme not only solves the Pseudo-Gibbs phenomenon to filter the signal effectively but also preserves the signal details to retain the diagnostic information. Meanwhile, the summed data processing method is advanced to realize the fast denoising. The simulation experiments on electrocardiogram(ECG) indicate that the proposed method can effectively and quickly separate signal from noise.**

Keywords-biomedical signal; Wavelet denoisin;, threshold function; multi-model method; summed data

I. INTRODUCTION

The biomedical signals are often corrupted by noise in their acquisition or transmission, which impedes further signal analysis and processing. As a result, first an appropriate signal preprocessing procedure for noise reduction is demanded for the successive biomedical signal processing. Since the biomedical signals are nonstationary signals, the high frequency interference can not be effectively filtered by the traditional time-domain or frequency-domain filtering methods.

A number of alternative time-frequency methods are now available for signal analysis. Of these, wavelet transform is especially valuable because of its ability to elucidate simultaneously local spectral and temporal information from a signal in a more flexible way by employing a window of variable width [1]. Thus, wavelet transform produces a time-frequency decomposition of the signal which separates individual signal components from noise to deniose the signals more effectively.There are three kinds of classical wavelet denoising methods including modulus maximum wavelet filtering, wavelet spatial correlation filtering and wavelet thresholding filtering which is most widely used.Considering the inherent limitations in existing threshold functions (TF), which might produce Pseudo-Gibbs effect in hard TF and constant deviation between reconstructed signals and the original signals in soft TF, the improvement of such threshold filtering methods is needed and promising. Meanwhile, the acceleration of implement of such method is preferred because of the computational effort required for processing of multi-level wavelet coefficients.

Hence, we propose a novel multi-model fast denoising method based on the Wavelet transform threshold denoising. The proposed denoising scheme not only solves the Pseudo-Gibbs phenomenon to filter the signal effectively but also preserves the signal details to retain the diagnostic information. Also, the summed data processing method is advanced to realize the fast denoising. The experiment results indicate that the proposed method can effectively and quickly separate signal from noise.

II. WAVELET TRANSFORM AND THRESHOLD DENOISING ALGORITHM

The wavelet transform (WT)[1] is a time-frequency analysis method by allowing arbitrarily high localization in time of high frequency signal features. The WT does this by having a variable window width, which is related to the scale of observation, a flexibility that allows for the isolation of the high frequency features. Rather, a large selection of localized waveforms $\psi(t) \in L^2(R)$ can be employed as long as they satisfy predefined mathematical criteria: $\int_R \psi(t)\mathrm{d}t = 0$. The wavelet transform of a continuous time signal x(t) is defined as: $T(a,b) = \int_{-\infty}^{+\infty} x(t)\psi_{a,b}(t)dt$, a is the dilation parameter and b is the location parameter of wavelet. The different choice of a and b can produce different time and frequency resolutions suitable for the analysis of nonstationary signals.

The basic idea of wavelet threshold denoising was proposed by Donoho[2]: the signals can be decomposed into high and low frequency subbands by WT, and the low frequency ones can be operated in this way repeatedly by the scale number. In fact, when representing a signal contaminated by additive "unstructured" noise using WT, some larger WT coefficients should mostly result from the signal components, while the noise part may, in general, contribute to almost all the small-valued ones. The fact stated above therefore leads to the idea of denoising a signal in wavelet domain[3].In threshold denoising, each coefficient is compared against threshold, if it is smaller than threshold, set to zero; otherwise it is kept or modified. Replacing those small noisy coefficients by zero and

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inversing WT on the thresholded result may lead to reconstruction with the essential signal characteristics and less noise[4]. It can be completed in three steps[2]:

1) decomposition: select appropriate wavelet basis function and scale number K, decompose noisy signals, obtain the low and high frequency coefficients V_k and W_k(k=1~K).

2) thresholding: estimate the noise threshold σ_k in each of coefficients W_k in each layer, then conduct the filtering processing on W_k according to the threshold function F_σ.

3) reconstruction: reconstruct denoised signal by using inverse WT on low frequency coefficients V_k in layer K and the filtered high frequency coefficients W_k from layer 1 to K.

III. The Denoising Method

A. The Estimation of Noise Threshold

Clearly, an appropriate estimation of the threshold is fundamental to the effectiveness of the denoising procedure. The high frequency coefficients after WT of noise in signals distribute symmetrically around zero and peak at zero, which can be described by the General Gaussian Distribution model[5]. Thus, the universal threshold σ proposed by Donoho[6] can be defined by Sqtwolog rules as:

$$\sigma = \delta\sqrt{2\ln(n)} \tag{1}$$

where δ is standard deviation of the noise, n is the number of WT coefficients in each level 1~m(1<m<K) of the signals.

B. The Threshold Function(TF)

The commonly used threshold functions include three kinds: hard threshold function, soft threshold function and semi-soft threshold function. The signals after the processing of the hard threshold function are not continuous at the threshold point which might produce Pseudo-Gibbs effect; although soft threshold function is of good overall continuity, the constant deviation exits, which affects directly the approximating degree of the reconstruction signals to the original signals; Compared with hard TF, the semi-soft TF lowers the degree of discontinuity at threshold point, while decreases the constant deviation compared with soft TF. Nevertheless, the semi-soft TF is single threshold TF so that it’s very sensitive to the estimation error that always exists in practice. To solve these problem, some improved threshold denoising algorithm[7-9] have been proposed, enhancing the denoising results and decreasing the influence of threshold estimation errors, but also had relative disadvantages respectively such as the complexity of calculation and so no. Thus, we propose a novel multi-model denoising method based on hard TF.

C. The Multi-Model Threshold Denoising

The original WT high frequency coefficients distribution is shown in Fig.2(a). As WT is good at energy compaction, the small coefficients are more likely due to noise and large coefficients are due to important signal features. The main idea of multi-model domain pattern (MMDP) proposed in this paper is to replace each coefficient with the maximum mean value among the candidate domain models to be compared with the threshold, producing the output after the thresholding step. This method can filter noise and pertain signal simultaneously.

1) Multi-Model Domain Pattern (MMDP)

At first, select pending data point P and set the steplength of the domain to be t , then produce t kinds of patterns around point P, as shown in Fig.1(take $t = 5$ for example). The rectangle in the figure defines the domain of patterns and the circled point is pending data P. Then, calculate the mean values of the different patterns respectively and replace point P with the maximum mean value as the characteristic value of P to act as the input of the thresholding step by being compared with the threshold.(by (6)).The advantages of MMDP are:

a) Optimization of noise reduction by eliminating the threshold estimation error

The estimation error always exists in practice. The magnitude of some noise points might outstrip the threshold, which would omit the thresholding process of those noise points. However, it is the domain mean value being comparing with the threshold as the characteristic value in MMDP. So, the average smoothing effect by calculating the mean value can lower the magnitude of those few unusual high noise points, guaranteeing the better denoising.

b) Extenuation of Pseudo-Gibbs phenomenon

To lessen the influence of Pseudo-Gibbs phenomenon in hard TF(part of coefficients of the signal is filtered by mistake taken as noise when the magnitude is lower than the threshold), the absolute maximum mean value in candidate patterns is used to replace original signal point P with small magnitude. In this way, by uplifting lower signal points with the help of averaging higher ones in the domain, it can be avoidable that smaller signal points are directly filtered and set to zero, destroying the useful signal information. As is shown is Fig.2(b,d).

c) Elimination of the constant deviation

In MMDP, we keep the entire reservation of those signal points greater than threshold, eliminating the costant deviation problem in soft TF(constant deviation between the detail coefficients of original signals and filtered coefficients leads to a difference between reconstructed denoised signals and the original signals.). As is shown is Fig.2(c,d).

2) The Estimation Of Domain Steplength

The idea of MMDP is to uplift the lower magnitude points using higher ones, hence, the domain interval should be around half pulse width. Also, the closer threshold and signal peak are, the more lower magnitude points needed to be uplifted and the larger steplength is demanded. Thus, the distance between threshold and signal peak is related to the estimation of domain steplength. The reasonable relationship should not be linear, meaning that the steplength could not change linearly as the distance change, but represent the spatial characters of the signal: the influence of distance to steplength decreases as the distance increases, namely, the rate of decrease of the influence slows down as the distance increases. Considering that the sigmoid function is asymptotically linear, smoothing and has high derivative value around zero, an estimation model for domain steplength is proposed in this paper, which can fit the signal spatial characters:

$$t = \lambda \times T \quad (2) \qquad \lambda = \frac{1}{1+e^{-\gamma(M/\sigma)}} \quad (3)$$

Amongst, Signal pulse width T: obtain the pulse width distribution by zero-crossing trait and calculate the mean value of the couples of the biggest width then apply it to be the Signal pulse width T (signal has larger width than noise because of it mainly belongs to low frequency); Signal peak magnitude M: compute the mean value M of all the magnitude of those points greater than threshold σ as the reference value of signal peak; Apply the ratio of M to as the independent variable input of the sigmoid function, γ is the control parameter for the independent variable range, since steplength is around $T/2$, the reasonable output of sigmoid function should be [0.4,0.6], setting γ=0.1 in this paper.

D. Summed Data Processing Method For Fast Denoising

To process each WT coefficient point requires calculation of mean values of m patterns in MMDP. In a length of N signal, to calculate all the points of the filtering output, the computational complexity just for the mean value is: N*m additions and N*m multiplication and division times. When N and m are both very large, the computational complexity will be increased dramatically. Therefore, the shortage of this method is high time complexity. In practice, we propose summed data processing method based on summed area to realize the fast denoising. The idea of summed data was proposed by Crow[10], in this paper, the mean value fast method is applied in MMDP. The summed data is defined as:

$$F(i) = \sum_{j=1}^{i} f(j), i \in [1, N] \quad (4)$$

N is the length of signal. A mean value fast method independent with steplength of the pattern can be implemented based on the summed data. By decreasing the computational complexity from N*m additions to N additions, the computing speed of the method has been improved dramatically.The steps are realized as follows:

(I) obtain the original signal f(i) and the scale number K. conduct WT to get the WT coefficient W_k in each level;

(II) compute the threshold of Wk in each level by (1), get the summed data W_{sk} during the calculation of variance of WT coefficients(not increase the computational complexity when getting the summed data)

$$W_{sk}(i) = \sum_{j=1}^{i} W_k(j), i \in [1, N] \quad (5)$$

N is the number of WT coefficients in level k(k=1-K).

(III) conduct MMDP processing on each WT coefficient, and compute the mean values of all candidate patterns using summed data:

$$mW_{Sk}(i) = \max_{j\in[1,m]} \frac{|(W_{sk}(i+m-j) - W_{sk}(i-j-1))|}{m} \quad (6)$$

Amongst, mW_{sk} is the WT coefficient output value after MMDP processing, Wsk is the summed data of WT coefficients and m is the pattern steplength.

(IV) select the maximum of absolute mean values in all patterns to performance as the final output comparing with the threshold.

IV. EXPERIMENTAL RESULTS AND ANALYSES

In this section, ECG is used to validate the superiority and effectiveness of the proposed methods.Matlab7.10 is used as the analysis tool. We selected the decomposed scale number K to 3. The following 4 types of methods are realized:

A: hard TF algorithmin B: semi-soft TF algorithm

C: algorithm proposed in [7] D: proposed in this paper

A. Quality Evaluation Standard of the Denoising Results

This paper selects the Signal Noise Ratio(SNR) and Correlation coefficient(CC) as the evaluation indicators adopted in many references.The larger SNR and CC are ,the better the denoising results are.

B. Experiments and Analyses of ECG

1) The Determination of Wavelet Basis Function

Considering the Wavelet filter length, orthotropic symmetry, linear phase and other factors according to article[11], the Bior2.2 Wavelet basis function has been found to obtain the best denoising effects when using Sqtwolog threshold in the filtering compared with other suitable candidate wavelet basis such as Sym3,Coif4 etc. Thus, Bior2.2 is the final wavelet basis for ECG.

2) Experimental Sample

American MIT-BIH database is used to validate superiority of proposed methods in this paper. At first, a “clean” ECG signal (Sampling rate is 360Hz, A/D is 11 bits) NO.103 is intercepted to be the original ECG signal. To testify the effectiveness of proposed method, the widely used noise simulation method[12] is applied by adding noise including industry frequency interference(simulated by 50Hz sine signal whose peak is 0.2 times the peak of QRS complex) and EMG interference(simulated by Gauss white noise, Mean 0 and variance 0.02 times the peak of QRS complex). So the noisy ECG signal is achieved and shown in Fig.3(b). 100 simulated noisy signals were generated. The denoising results are measured by the average value of the denoising indicators in the 100 experiments. The evaluation of denoising effects using 4 different methods is in TABLE I, and ECG denoising effects are shown in Fig.3.

C. Analyses and Comparison

The results of the experiments indicate that: in the comparison in visional effect and detail observation, the denoising results, SNR and CC of the proposed method in this paper are appreciably prior to those of method A(hard TF), B(semi-soft TF) and C. Method C is proved to be better than the traditional improved TFs like VisuShrink, SureShrink, BayesShrink[7] in removing noise. Therefore, the proposed method in this paper is better than hard TF, semi-soft TF and traditional improved TFs, possessing better approximate degree between original signals and reconstructed signals.

V. Conclusion

In this paper, we analyses the problems in hard and soft TH method and propose a novel multi-model fast denoising method based on the Wavelet transform threshold denoising. The proposed denoising scheme not only solves the Pseudo-Gibbs phenomenon to filter the signal effectively but also preserves the signal details to retain the diagnostic information. Meanwhile, the summed data processing method is advanced to realize the fast denoising. The experiment results indicate that reconstructed signals using the proposed method suppress the noise effectively and pertain the trait of original signal, also the SNR, CC and visional effect have been greatly improved. This method is proven to separate signal from noise effectively and quickly.

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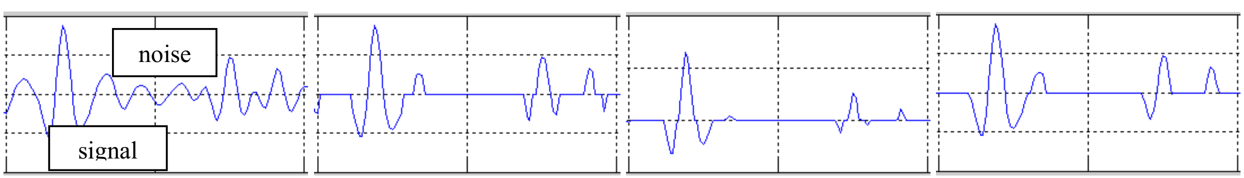
TABLE I. The Denoising Results Of Different Methods On ECG

	Method A	Method B	Method C	Method D
SNR	12.4298	13.1650	13.4062	14.1855
CC	0.9715	0.9756	0.9769	0.9805

(a)pattern 1 (b)pattern 2 (c)pattern 3 (d)pattern 4 (e)pattern 5

Figure 1. Schematic diagram of MMDP(t=5)



(a)original WT coefficients (b)Pseudo-Gibbs phenomenon in hard TF (c)constant deviation in soft TF (d)WT coefficients after the MMDP processing

Figure 2. Diagram of WT coefficients after different methods

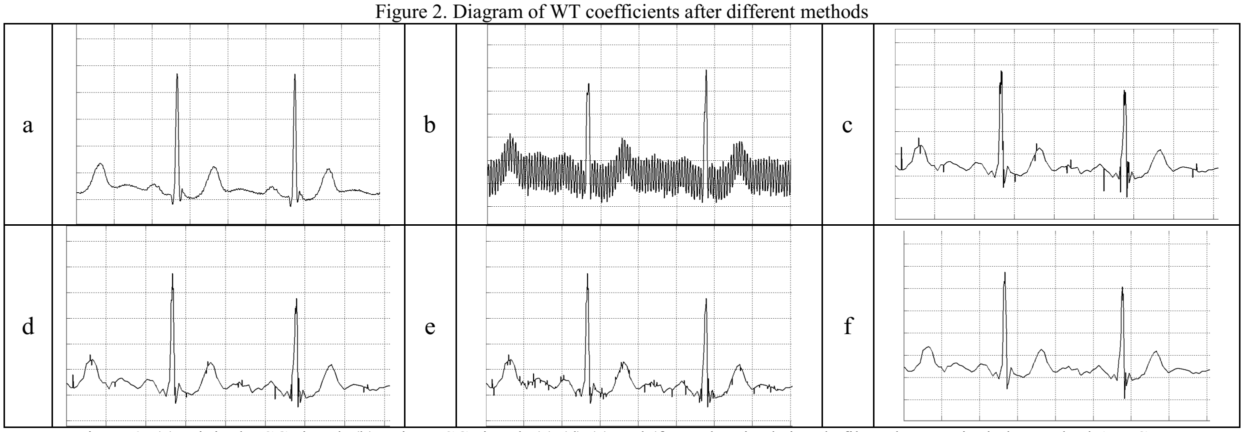


Figure 3. (a) original ECG signal; (b) noisy ECG signal; (c),(d),(e) and (f) are denoised signals filtered respectively by method A,B,C,D